1927HarCi.301....1P

HARVARD COLLEGE OBSERVATORY

CIRCULAR 301

ON METHODS IN STELLAR SPECTROPHOTOMETRY

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1. The problem of the measurement of the intensities of spectrum lines has been under investigation at Harvard for the past two or three years. Preliminary discussions of the method have already been published,[1] and results for several spectra have also been given in Harvard Reprint 28. It is proposed to publish a more extended set of results in the near future, and the present paper contains an account of the amplifications and tests of method that have resulted from further study of the problem. The measurement of line intensities now appears to have been reduced to a satisfactory working basis.

2. The fundamental problem consists in setting up an accurate photometric scale on the spectrum plate, and relating with this scale the intensity differences from background to line center. Although the calibration curves for different plates of the same kind are roughly similar, their differences, as is well known, are far too great to permit accurate direct intercomparison, and therefore the scale must be set up independently for each plate. In Section 6 the qualities of the various plates used in the present research will be compared.

The general method of setting up a photometric scale by means of diaphragms placed over the objective prism has already been discussed in Harvard Bulletin 805, and the methods of measurement and reduction have been outlined in Harvard Reprint 28; they will not be further described. A second method of setting up a scale on the plate, described in detail in Section 2 of the present paper, consists in substituting for the spectra of one star, photographed with various diaphragms, single spectra of several stars with known magnitude differences. The two methods are clearly alike in principle.

The Reduction Curves. In the original treatment of the problem a separate reduction curve was drawn for each wave length, thus eliminating any inaccuracy that might result from the different sensitiveness of the emulsion to light of different colors. The color sensitiveness of the plate enters only as a second order correction, however, since the method used is differential, the center of a line being compared

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with the continuous background at the same wave length. The use of a separate reduction curve for each wave length is only necessary if the *gradation* of the plate is different for light of different colors — that is, if the increase of density of the image on the plate for a given increase in the intensity of illumination is different for different colors. That the gradation shall not be a function of wave length is a far less drastic requirement than the requirement that the sensitivity shall be independent of wave length. To the order of accuracy attained in the work under discussion the gradation appears indeed to be approximately constant for any one plate, as may be seen, for example, from Figure 1; for all the wave lengths represented, the curves corresponding to the same increments in the intensity of the incident light are roughly parallel.

It is evident that if the departure of the curves from parallelism for different wave lengths is of the same order as the accidental error of any one point on a curve, more accurate values of intensity will be obtained from a reduction curve made by combining all available curves for individual wave lengths, than from the single curves themselves.

The accuracy of a combined reduction curve must be regarded entirely as a matter for experiment, and in practice it appears to be fully satisfactory, as illustrated in Figures 1 and 2. The figures contain a set of reduction curves drawn for different wave lengths, and also the combined reduction curve made by displacing the various curves parallel to the axis of abscissae. The typical reduction curves shown in the two figures illustrate the accuracy with which the individual curves run parallel one to another.

Besides increasing the accuracy of the results, the combination of the curves for all wave lengths into a single reduction curve serves to detect serious deviations from standard conditions for individual spectra. Several cases have been encountered in which the combined reduction curve showed that one of the spectra on a plate was affected by some disturbing factor that made it too bright or too faint (an effect that could be produced by irregular driving or by variations of atmospheric transparency). Inaccuracies arising from effects of this sort would not usually be detected when a separate curve was used for each wave length, and their elimination is one of the greatest advantages of the method of combined curves.

Combined reduction curves of the kind described have been used throughout the present investigation, and some results based upon them have already been published.[2] It is of interest to note that a closely similar method of combining reduction curves, applied to laboratory spectra, has been independently developed and described by Dorgelo [3] and by van Milaan.[4]

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Apertured Spectra as Standards. The reduction curve reproduced in Figure 1 is based upon six spectra of Capella, secured on the same plate with apertures 1, 2, 3, 4, 5, and 6, corresponding to magnitude intervals of 0: 0.375: 0.750: 1.125: 1.500: 1.875. For fainter stars the length of exposure required makes it impossible to secure so many spectra for calibration of the plate; in most cases four spectra are used.

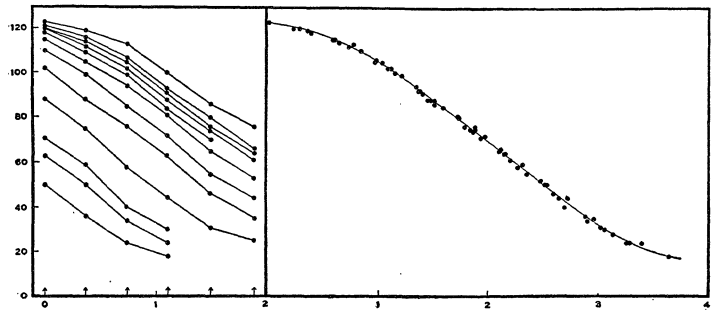

FIGURE 1.— Reduction curve derived from six apertured spectra of Capella. Ordinates are galvanometer deflections in half millimeters; abscissas are intensities in stellar magnitudes. The material used for constructing the curve is in the section to the left. The arrows on the lower margin indicate the magnitude steps between the various spectra, which were photographed through apertures with areas in the ratio 5.6 : 4 : 2.8 : 2 : 1.4 : 1. Points relating to any one wave length are connected by straight lines; they represent a number of arbitrarily chosen positions in the spectrum. The section on the right contains the reduction curve made by moving the points corresponding to any one wave length parallel to the axis of abscissas.

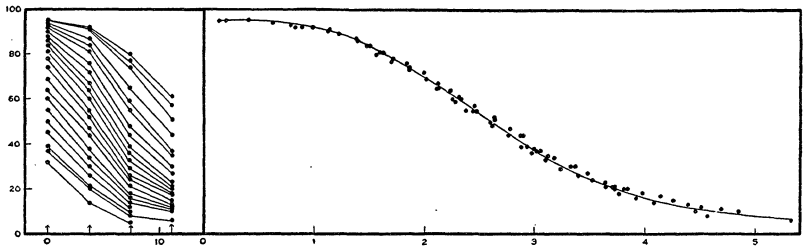

FIGURE 2.— Reduction curve derived from four apertured spectra of Procyon. Ordinates are galvanometer deflections in half millimeters; abscissas are intensities in stellar magnitudes. The diagram is similar in form to Figure 1. The magnitude steps between the four spectra correspond to apertures in the ratio 16 : 8 : 4 : 2.

The question arises how many spectra are necessary for the accurate establishment of an intensity scale. Figure 3 illustrates a test of this matter. Two of the six spectra of Capella represented in Figure 1, with an intensity difference of $0^m.375$, were chosen as representing the most unfavorable case possible, and a reduction curve was constructed from them. The curve is indistinguishable from the curve made with the aid of six spectra. Thus it is possible to obtain an accurate calibration of a plate by means of only two apertured spectra. The line intensities derived from such a plate would be less accurate than intensities derived from six spectra, only because of the smaller number of values contributing to the average. The possibility of using only

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two apertures in calibrating a plate permits the extension of the work to much fainter stars than was at first thought possible. For faint stars, however, it is usually more convenient to calibrate the plate by photographing a bright star with a series of diaphragms, and putting the faint star on the same plate. The star used for calibration of such a plate receives an exposure different from that for the fainter star, and is usually photographed at a different altitude. Whatever errors may thus be introduced are counterbalanced by the greater accuracy of the reduction curve, when the differences of altitude and exposure are not too great.

The Pleiades as Standards. The scale set up by means of diaphragms for the calibration of a plate is subject to certain sources of inaccuracy, of which the most

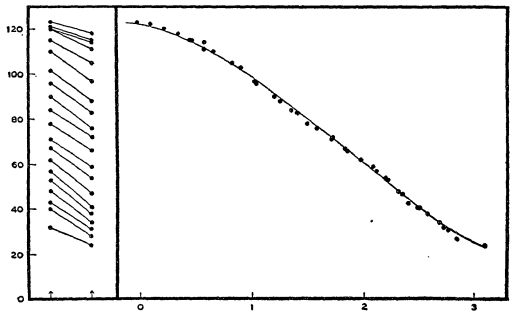

FIGURE 3.— Reduction curve from two of the spectra of Capella illustrated in Figure 1. The arrangement is as in that figure. The ratio in aperture for the two spectra is 1.4 : 1. The curve drawn through the points is the curve obtained in Figure 1 from the six spectra, moved along the axis of abscissas until it coincides most nearly with the plotted points. The closeness with which these points fall on the curve illustrates the accuracy that can be obtained with two apertured spectra.

important is the changing transparency of the atmosphere. This disturbing factor can often be detected, as pointed out above, when the method of apertures is used, but is not easily eliminated, and might, in some cases, pass unnoticed. A progressive dimming of a star by haze would, for example, distort the reduction curve, decreasing or increasing its steepness according as successive apertures increased or decreased in size. A uniform effect of this kind would probably only appear when the results from several plates were compared. Besides variations of transparency, irregularities of clock rate, and changes due to refraction (especially for faint stars, for which long exposures are necessary) introduce inaccuracies for apertured spectra.

In order to eliminate inaccuracies of this sort, the spectra of the Pleiades have been used for setting up a scale of intensities. These stars are close enough together to be photographed simultaneously on the same plate. In treating their spectra as equivalent to a set of apertured spectra of the same star, with intensity differences

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equal to their differences of apparent magnitude, it is assumed that the distribution of intensity in the continuous background is sensibly the same for all. The assumption appears plausible from the fact that the six bright Pleiades that give spectra not superposed on those of other stars (H.D. 23288, 23302, 23338, 23408, 23480, 23630) are all of Class B5, and that their color indices are sensibly equal.[5] The possibility of setting up an accurate calibration curve with their aid must again be regarded as a matter for experiment. One of the curves so made, and reproduced in Figure 4, evidently compares favorably with curves constructed from apertured spectra. The continuous spectrum, and other features of the Pleiades, will be discussed in a later paper. The use of these stars for plate calibration has the advantage that all spectra are photographed under identical conditions of sky and clock driving.

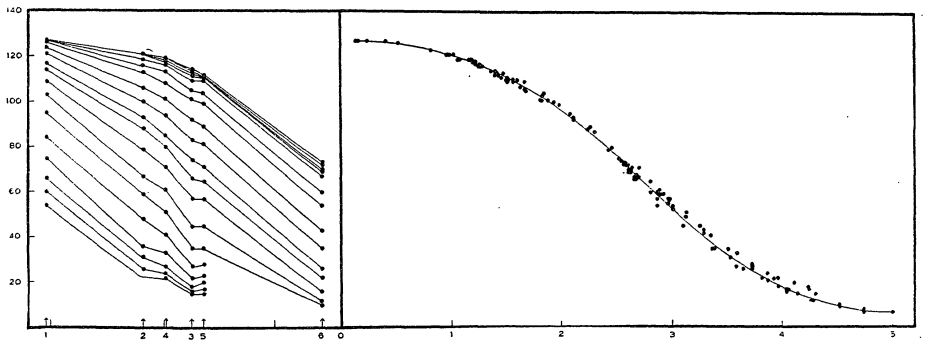

FIGURE 4.— Reduction curve derived from six of the brighter Pleiades, all of Class B5. Arrangement and coordinates are as in Figure 1. The numbered arrows refer to the stars used, which are tabulated below with their adopted magnitudes —

1. Alcyone	2.96	4. Maia	4.02
2. Electra	3.81	5. Taygeta	4.37
3. Merope	4.25	6. Celaeno	5.43

As a result of the success with which a reduction curve can be obtained from two apertured spectra, it may also be considered legitimate to use two neighboring stars of similar spectral class for the calibration of a plate. Results thus obtained have recently been published[6] for 29 Canis Majoris and 30 Canis Majoris, stars of Class O that are presumably of nearly equal temperature.

Stars that are to be used for calibration of a spectrum plate must agree in spectral class within very narrow limits in order that the distribution of energy in the backgrounds may be similar. The Pleiades are the most suitable group of stars for the purpose, but undoubtedly other suitable groups exist, notably in crowded regions near the Milky Way. Any group of stars that gives a reliable reduction curve can be used to calibrate a plate on which other stars, regardless of their spectral class, are photographed. This method is used, for example, in an investigation of Mira Ceti that is now in progress — the Pleiades and Mira being photographed upon the same plate.

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Calibration by varying Exposure Time. A comparison of the results derived from objective prism and slit spectra is of importance, especially as the results that have been published by workers with slit spectrographs suggest that lines are blacker than observed at Harvard.[7] The method of aperturing, at least in the form used at Harvard, is inapplicable to slit spectra, and has usually been replaced by a variation of exposure time.

To provide a comparison between the method of apertures and the method of varying exposures, an investigation of Sirius has been undertaken. Two plates of the star were made, one with a series of apertures, the other with a series of different clock rates. The second plate was reduced to "effective apertures" by means of the Schwarzschild exponent p, according to the formula $\triangle m = 2.5\ p\ (\log r_1 - \log r_2)$ where r_1 and r_2 are the clock rates, assumed to vary directly with the driving load.[8]

The accompanying table contains a comparison of the line depths deduced by the two methods. The method of varying the clock rate gives results of the same order as the method of apertures.

TABLE I

LINE DEPTHS FOR SIRIUS

Line		Apertured	Rated
Hβ	4861	59	53
Hγ	4340	72	62
Hδ	4101	74	67
Hε	3970	74	69
Hζ	3889	76	74

A single value of 0.8 was used throughout for the constant p. The uncertainty of the value of p to be used for the particular emulsion and wave lengths in question is as great as the differences between the results obtained by the two methods.

The method of varying exposures has been avoided throughout the main investigation, because of the well-known difficulties it involves, such as the variation of the Schwarzschild exponent with wave length. The results of a comparison of objective prism and slit spectra of the same star, taken under similar conditions, will be published elsewhere.

3. Of the numerous possible sources of error, the following are probably of principal importance: variations of seeing, poor focus, irregularity of clock rate, Eberhard effect. The first three would tend to reduce the measured line depths, the last to increase them. The possibility that an Eberhard effect plays an important part in the results has been examined in Harvard Reprint 28 and rejected; it will not be discussed further.

Effect of Seeing. Poor seeing would tend to flatten the contour of a line by dis-

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placing the image periodically on the plate as the star twinkles. This effect could not be avoided by the most careful following, as the changes are too rapid. Considerable care has been used in examining the size of errors resulting from this cause.

Agreement between the results obtained on different nights for the same star shows at once that the effect of varying seeing is not a large one. The agreement between different plates is less good for stars with narrow lines (γ Cygni) than for the Pleiades, which would be expected if the differences from plate to plate were chiefly the result of varying seeing.

To examine a spectrum under changing conditions of seeing, several stars have been photographed at intervals as they rise, cross the meridian, and set. The depths of the lines, derived from light that has passed through a series of different thicknesses of atmosphere, give an idea of the total effect of seeing upon measured line depth. The experiments did not reveal any change greater than two per cent of the background between spectra photographed at altitudes from 27° to 75°.

It is possible to see how far atmospheric irregularities and uneven clock rate affect the results from the spectra themselves. All the spectra show more or less longitudinal streaking, sometimes regularly spaced, and clearly due to a periodicity in driving mechanism, sometimes irregular, and resulting from atmospheric variations. Figure 5 illustrates the different types of streaking. It reproduces microphotometer tracings made by analyzing the spectra perpendicular to their length, in a region as free as possible from lines. The first curve represents an analysis of a spectrum of Sirius photographed by trailing the image without the driving clock. Any streaking that occurs in this spectrum must be a pure sky effect; from the period of the variations it appears that they represent the ordinary twinkling of the star. Most of the spectra analyzed are widened so much more slowly than this one that the short period variations due to twinkling are too close together to be important. The second is taken from a plate that shows regular streaking, due to a periodic driving error. The resulting decrease in measured line depth is not large. The width of the analyzing beam of the microphotometer used in the experiment is 0.04 mm, and its length is about 0.5 mm. Thus, when the spectrum is analyzed in the usual way, the integrated longitudinal strip is about ten times as wide as the strip when the spectrum is analyzed crosswise. The errors, which amount to about $0^m.10$ in the diagram, are thus reduced, in the process of analysis, to quantities of the order of one hundredth of a magnitude.

Effect of Focus. In Harvard Reprint 28 the effect of various focal settings upon the measured line depths was examined for the 16-inch refractor, with which all the spectra that are discussed there were photographed. These measures showed the

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differences of depth that were produced by changing the focus, but took no account of the fact that with a refractor only a small part of the spectrum is accurately in focus at one time, if it is photographed on a flat plate.

Spectra taken with the 24-inch reflector are in focus at all wave lengths, and a comparison of results obtained with it and the 11-inch refractor, which gives nearly

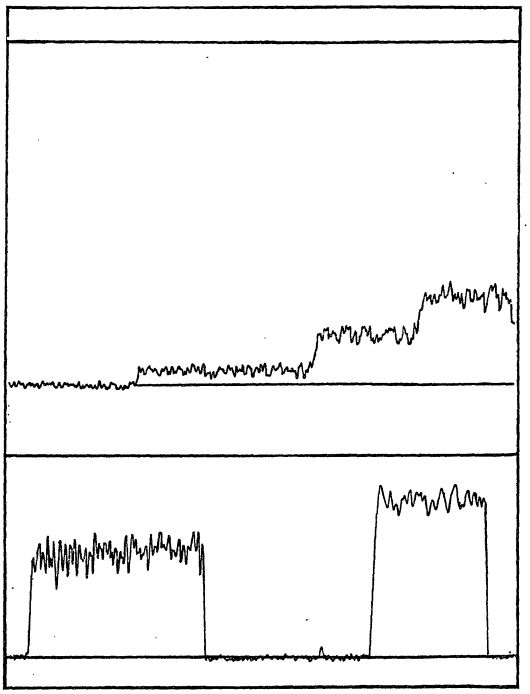

FIGURE 5.— Microphotometer tracings made by analyzing spectra perpendicular to their length, to illustrate the longitudinal streaking. The upper part of the figure represents the spectrum of Sirius trailed across the plate without driving. The four levels, reading from the left, represent respectively clear film, and Sirius trailed once, twice, and three times. The line for "darkness" is shown at the top of the diagram. The irregularities that exceed those shown by clear film, and that are therefore not due to plate grain, represent the twinkling of the star. Their period is of the order of half a second. The lower part of the diagram represents two spectra of Sirius conspicuously streaked by a driving error with a period of about twenty seconds. When the spectrum is analyzed, about five of the longitudinal streaks are integrated by the analyzing beam. "Darkness" and "clear film" are marked by horizontal lines.

the same scale, measures the effect of focus for plates taken with the latter. A similar 15° prism is used on both instruments.

The following tabulation gives the mean measured line depths for H.D. 23302, 23338, 23408, and 23630 (Pleiades), derived from 1-prism plates taken with the refractor and the reflector, respectively.

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TABLE II

MEAN LINE DEPTH

Line		Reflector	Refractor	Difference
Hβ	4861	34	31	+ 3
Hγ	4340	48	44	+ 4
Hδ	4101	50	50	0
Hε	3970	54	55	− 1
Hζ	3889	53	54	− 1
Hη	3835	57	52	+ 5
Hθ	3798	57	52	+ 5
Hι	3771	56	47	+ 9
Hκ	3750	51	37	+14
Hλ	3735	42	31	+11
Hμ	3722	36	22	+14
Hν	3712	32	16	+16

4. The accuracy of the derived intensities of lines may be measured by the comparison of several plates of the same star, preferably of different photographic densities, and taken under dissimilar conditions. Most of the spectra discussed in

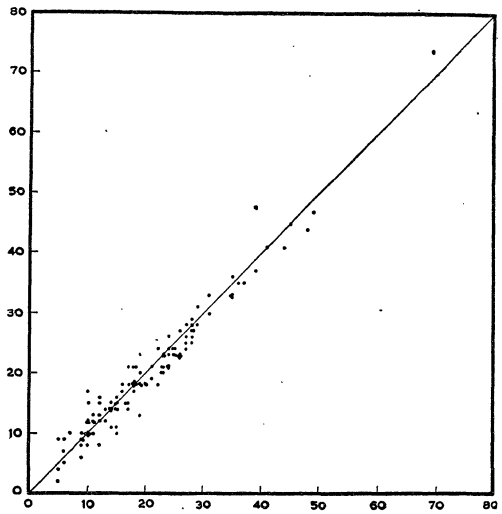

FIGURE 6.— Comparison of results derived from two 1-prism plates of Capella, of very different densities. Ordinate and abscissa are line intensities, reduced to percentages of background, for plates C 18583 and C 18584 respectively. Each of the plates has six spectra, taken with apertures 1, 2, 3, 4, 5, 6, and 6, 7, 8, 9, 10, 11, for the respective plates; the interval in magnitudes between apertures is 0.375. The last spectrum on C 18584 was too faint to be analyzed. The straight line has the slope required for equality; the points show no systematic deviation from it.

the present series of notes have been measured on at least two plates, and a few typical cases are chosen to illustrate the agreement that exists for the results from various plates. A comparison between two plates of Capella, differing greatly in density, is shown in Figure 6. The agreement for the two plates is satisfactory. In

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Table III the errors of the values of individual line intensities are indicated. They represent the mean of the deviations of any individual line from the mean intensity for that line. These mean errors have been evaluated separately for the intervals of intensity at the heads of the columns. The error appears to be constant, rather than proportional to Dl.

TABLE III

MEAN ERRORS OF LINE INTENSITIES

Star	Number of Prisms	Number of Plates	Number of Lines	Intervals in D1 0–10	10–20	20–30	30–40	40–50	50–100
Capella	1	2	158	0.8	1.0	0.9	1.5	2.0	4.9
γ Cygni	1	2	132	0.7	1.9	1.2	1.4	1.3	1.5
	2	2	171	2.0	1.6	1.4	1.5	1.0	2.6
Pleiades	1	9	519	..	3.1	2.8	4.3	3.7	3.2

It should be mentioned that some stars systematically show discordances that are greater than would be expected from the reduction curves, or from the good agreement that is obtained for other stars. The outstanding example is α Cygni, for which the line depths measured from different plates may frequently differ by more than half a magnitude. This variability, which is under investigation, may be the effect of poor seeing or bad focus, to which the depths of such narrow lines are very sensitive; but the possibility remains that it is real.

5. In a forthcoming paper [9] it is verified that the measured depth of a spectrum line is a function of the resolving power used in forming it, and that for all lines hitherto investigated at Harvard the line width instrumentally produced considerably exceeds the true line width. A test of the accuracy of this statement, and a measure of the degree to which resolving power affects the measured line depths, is afforded by intercomparing the results obtained with one 6° prism and with one, two, and three 15° prisms.

TABLE IV

LINE DEPTHS FOR FOUR PLEIADES STARS, VARIOUS RESOLVING POWERS

Line	6°	15°	30°
Hβ	25	36	40
Hγ	38	44	52
Hδ	41	51	55
Hϵ	33	55	58
Hζ	30	54	58
Mean	33	48	53

More extended work has shown that the statement recently published by one of the writers [10] that there is no systematic difference between spectra made with one and two 15° objective prisms, is incorrect; when the resolving power is increased, the measured depths are systematically, but very slightly, increased.

A comparison between the line depths obtained for γ Cygni with one and two

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objective prisms is contained in Figure 7. The depths of the lines are converted from stellar magnitudes to percentage light losses. The correlation is linear; the best line passing through the points runs approximately through the origin, and the line depths derived from two prism plates are systematically greater than those from one prism plates. The factor introduced in the line depth by approximately doubling the resolving power is +1.16. A considerable dispersion is evident from Figure 7; the deviations from the mean are considerably larger than the probable errors of the individual points in the two coordinates, as may be seen by reference to Table III. This must be the result of differences of line quality, and is an indication that the assumption of similar contours for all lines is not entirely accurate.

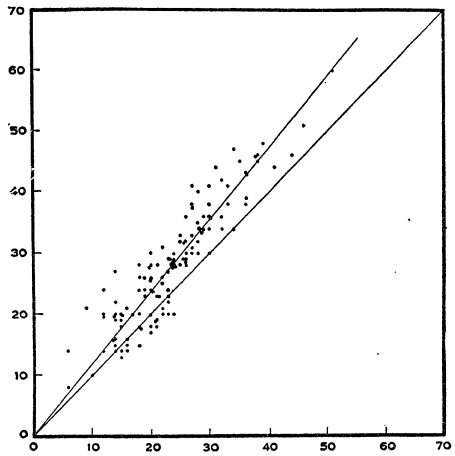

FIGURE 7.— Comparison of results derived, from plates taken with one and two objective prisms, for γ Cygni. Ordinates are mean intensities of lines, expressed in percentages of background, from two plates taken with two prisms. Abscissas are corresponding quantities derived from two plates taken with one prism. The diagonal line represents equality. The upper line is drawn to represent the relation between the values obtained with the two dispersions; its inclination to the diagonal line represents the systematic correction to be applied to the one-prism results to reduce them to the two-prism scale. The deviations of the individual points from the upper line are greater than the deviations from one another of the results derived from two plates made with the same dispersion.

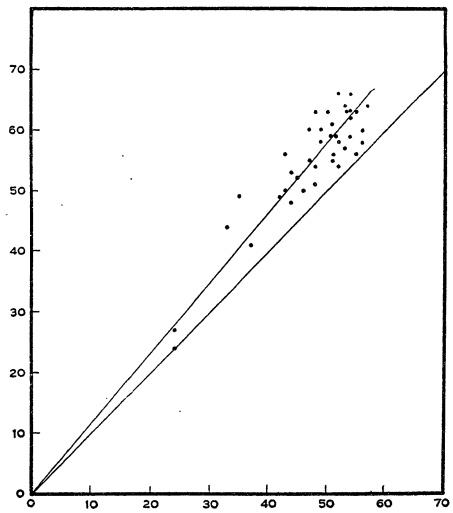

FIGURE 8.— Comparison of results derived from spectra of the Pleiades taken with one and two objective prisms. Ordinates are two-prism values and abscissas are one prism values, both expressed in percentages of the intensity of the background. The lower line represents equality; the upper line represents the correlation between the values obtained with the two dispersions. The inclination of the two lines to one another represents the systematic correction to be applied in converting one-prism results to the two-prism scale. The inclination is sensibly the same as that shown in Figure 7, although the qualities of the lines of γ Cygni and the Pleiades are very different.

The comparison given in Figure 7 applies to a star with sharp and narrow lines. A similar comparison, for the lines of the Pleiades, is contained in Figure 8. The correlation is similar to the one found for γ Cygni. The factor, 1.16, relating values of Dl obtained with one and with two prisms, is sensibly the same for the two figures.

6. In concluding this synopsis of method, it will be of interest to illustrate the characteristic curves of a number of the plates that have been used. These curves,

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arbitrarily reduced to an amplitude of 100, are illustrated in Figure 9. Their actual amplitudes, measured in half millimeters of galvanometer deflection, range from 180 to 40. Figure 9 illustrates both the rough similarity of the curves for all plates, and the conspicuous differences of slope from one plate to another.

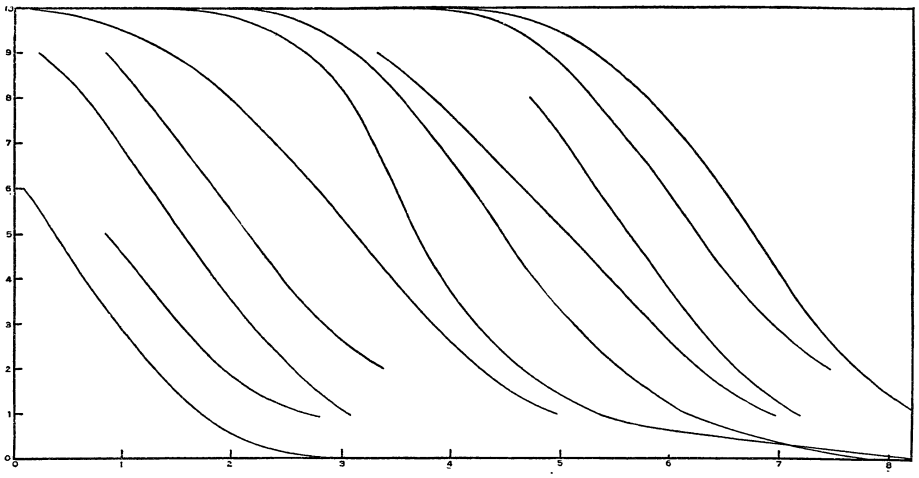

FIGURE 9.— Reduction curves for eleven of the plates used in the present investigation. All are Presto plates (reading from the left, C 18584, C 18557(1), C 18630, C 18557(2), C 18598, C 18601, C 18514, C 18568, C 18676, C 18602, C 18614). Ordinates are percentages of total galvanometer deflection; abscissas are intensities of incident light in stellar magnitudes. Each of the curves represents the whole range of blackening for the corresponding plate; C 18601 was completely burned out in the more heavily exposed portions, whereas C 18584 was somewhat lightly exposed, and never cut off more than sixty per cent of the analyzing beam in the microphotometer.

SUMMARY

1. The method of spectrophotometry in use at Harvard is discussed in the present paper.

2. A photometric scale is set up on each plate by means of the stars themselves. Reduction curves are constructed for the whole plate by the combination of the reduction curves for individual wave lengths. Such reduction curves are of satisfactory accuracy.

3. Sources of error are discussed, and the size of the errors is examined.

4. The accuracy of the results is tested by the agreement between the values derived from different plates of the same star.

5. The effects of changing resolving power are empirically evaluated.

6. Characteristics of a number of the plates used in the investigation are graphically compared.

* National Research Fellow.
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FEBRUARY 14, 1927

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